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Aerodynamic Drag Reduction for Satellites in Low-Earth Orbits

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It has been suggested in Ref. 1 that, by ejecting molecules directly upstream from the entire face of a satellite, it is possible to reduce the drag on a satellite in low-Earth orbit and hence maintain orbit with a total fuel mass (for forward ejection and conventional reaction rockets) less than the typical mass requirements of conventional rockets. An analytical analysis is presented here, as well as Monte Carlo simulations, that indicates that to significantly reduce the overall drag on the satellite, collisions between the freestream and ejected molecules must occur at least two satellite diameters upstream. This can be achieved if the molecules are ejected far upstream from the satellite's surface through a sting that projects forward from the satellite. Using some estimates of what would be feasible sting arrangements, we find that the drag on the satellite can be reduced to such an extent that the satellite's orbit can be maintained with a total fuel mass of less than 60% of that required for reaction rockets alone. Upstream ejection is only effective in reducing the drag for freestream Knudsen numbers less than approximately 250.

Nomenclature

A	= area of disk/satellite surface
A_r	= area of rocket exhaust
A_2	= area of ejection nozzle
C_D	= drag coefficient, drag force/$\frac{1}{2} m_1 n_1 U_1^2 A$
C_{D0}	= drag coefficient in the absence of upstream ejection
C_{Dfm}	= free molecular drag coefficient
D	= diameter of disk/satellite
d	= diameter of sting
f	= velocity distribution function
g	= relative speed of molecules
Kn	= Knudsen number, λ_∞/D
l	= length of sting
m	= mass of molecules
n	= number density
R	= gas constant
R_f	= mass flow rate ratio, $(m_2 n_2 U_2 A_2)/(m_1 n_1 U_1 A)$
S	= speed ratio, $U/(2RT)^{1/2}$
S^*	= $[(\omega_1 + \tau\omega_3)^2 + \omega_2^2]^{1/2}$
T	= temperature
T_S	= temperature of disk surface
U	= stream velocity
u	= molecular x velocity component
V	= rocket exhaust velocity
v	= molecular y velocity component
w	= molecular z velocity component
α	= center angle of ejection from sting
β	= angle between U_1 and v
δ	= angle of divergence of ejected molecules
ζ	= $w/(2RT)^{1/2}$
η	= $v/(2RT)^{1/2}$
λ	= mean free path, $(2^{1/2}\pi n \sigma^2)^{-1}$
ξ	= $u/(2RT)^{1/2}$
σ	= molecular diameter
σ_{12}	= $(\sigma_1 + \sigma_2)/2$
τ	= $(T_2/T_1)^{1/2}$
ω_1	= $S_1 \sin\beta$
ω_2	= $S_1 \cos\beta$
ω_3	= $U_2/(2RT_2)^{1/2}$

Subscripts

r	= reaction rocket molecules
1	= class 1 molecules (freestream)
2	= class 2 molecules (ejected)

Superscript

(1)	= first order

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Introduction

SATELLITES in low-Earth orbits experience significant aerodynamic drag as they move through the low-density atmosphere. To avoid decay of the orbit, rocket thrusters eject propellant in the opposite direction to the satellite motion to counteract this drag. Any means of reducing the mass of propellant required is welcome since it enables either the payload or the life of the satellite to be increased.

Stalker[1] suggested that, by ejecting a jet of molecules forward from a satellite surface, it is possible to deflect atmospheric molecules from the path of the satellite and, in so doing, to decrease the drag by more than enough to compensate for the reaction caused by the forward ejection. This was referred to by Stalker as "molecular sweeping." Forward ejection of a gas from satellites is not new. Work has been done on the protection of satellite-borne experiments and protection of cryogenic optics in space infrared telescopes using a purging gas flow.[2] That is, by ejecting a stream of molecules in front of a satellite, the ejected gas acts as a barrier to oncoming high relative speed molecules. However, the momentum considerations of forward ejection have not been examined in detail. Here we explore the possibility that molecular sweeping may provide a more efficient way of overcoming the drag on satellites than the use of normal reaction rockets alone. We use a simple analytical model in which we assume that all collisions take place one mean free path from the satellite, and we use Bird's[3] direct simulation Monte Carlo (DSMC) technique to check some of the results of our analytical analysis.

Various arrangements of molecular sweeping jets were studied, and it was found that if the protective molecules can be ejected sufficiently far upstream of the satellite's surface, through a "sting" that projects forward from the satellite, then the drag on the satellite can be reduced for Knudsen numbers less than 250, that is, in the transition regime as free molecular flow is approached. Knudsen numbers less than 250, for typical satellite dimensions, represent very low-Earth

250, for typical satellite dimensions, represent very low-Earth orbits that are generally not used at present but may become more attractive if the fuel savings indicated in our analysis are realized in practice. Although we have not attempted to design such a sting in detail, we do show that, for some reasonable estimates of what may be feasible sting arrangements, the satellite's orbit can be maintained with a total fuel mass (for forward ejection and conventional reaction rockets) considerably less than that required for a typical reaction rocket.

Molecular Sweeping Problem

Stalker's analysis[1] of molecular sweeping relied on the assumption that the collisions between ejected and freestream molecules occurred in a narrow zone, the location of which could be varied by altering the number flux of ejected molecules. Our analysis takes into account the expected mean free path of ejected molecules before they undergo a collision with a freestream molecule. A first-order approximation to this mean free path is derived in the Appendix.

The satellite is represented as a flat disk, at rest in the chosen reference frame, aligned normal to an oncoming freestream (Fig. 1). The ejected molecules are represented as a uniform jet, ejected directly upstream from the entire face of the disk with mean velocity U_2 and temperature T_2. Taking a typical point of collision at one mean free path from the disk on the x axis, we can calculate the solid angle subtended by the disk from this point. For hard sphere scattering, the distribution of the postcollision relative velocities is isotropic in the center of mass reference frame, and so the percentage of molecules scattered into the solid angle can be found. The number flux of ejected molecules is assumed to be equal to the incoming number flux of freestream molecules, that is,

$$n_2 U_2 = n_1 U_1 \quad (1)$$

and every ejected molecule is assumed to collide with a freestream molecule. Complete thermal accommodation is assumed for all molecules striking the disk. With these assumptions, the drag on the disk due to impinging molecules can be found. The total drag is then calculated by including the reaction from the forward ejection.

Investigation of different combinations of the molecular mass ratio (ejected/freestream) and the ejected speed of molecules revealed that the total drag coefficient on the disk is consistently increased above the free molecular (collisionless) drag coefficient C_{Dfm}, which is an upper estimate of the aerodynamic drag coefficient in the absence of upstream ejection. A sample result is shown in Fig. 2 for a Knudsen number of 1.6 and a temperature ratio T_S/T_1 of 1.0. Note that by the assumptions incorporated in the analytical model, the drag due to impinging freestream molecules is equal to the drag due to ejected molecules scattered back onto the disk when the freestream and ejected molecular masses are equal. The total drag is found to be insensitive to changes of the ejected speed ratio and mass ratio, since any reduction in the rate that incident molecules hit the disk is overcome by the increased backward reaction from the ejection. The degree of protection of the disk from "contamination" by freestream molecules is also indicated from the results of the analytical method. It is found that for larger mass ratios (heavier molecules used for ejection) the proportion of the drag due to freestream molecules impinging on the disk is reduced, that is, the satellite is more protected.

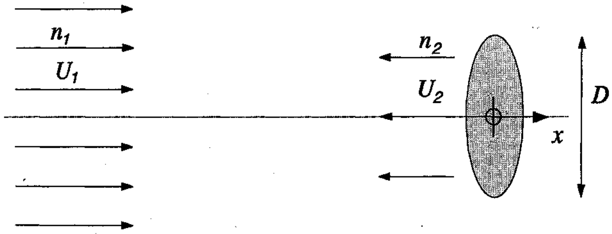

Fig. 1 Schematic of domain of upstream ejection process.

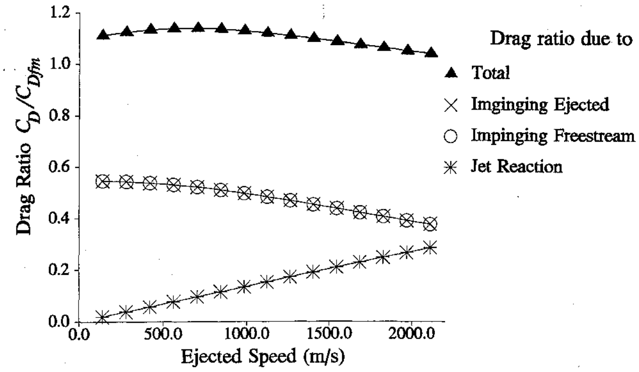

Fig. 2 Drag coefficients on circular disk incorporating molecular sweeping. Results from analytical model for a freestream speed of 7.6 km/s, speed ratio of 16.5, Knudsen number of 1.6, $m_1 = m_2$, $T_1 = T_2 T_s$, and $n_1 U_1 = n_2 U_2$; $C_{D_{fm}}$ = drag coefficient for no ejection = 2.11 (free molecular value).

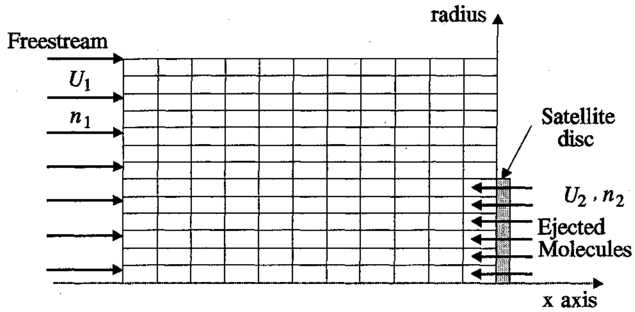

Fig. 3 Schematic of the Monte Carlo simulation flowfield.

DSMC Simulation of Molecular Sweeping

Our approximate "first collision" analysis has given a different answer for the total drag on a satellite caused by upstream ejection than was found by Stalker, and so further investigation was made using Bird's DSMC method.[3]

Direct Simulation Monte Carlo Method

The DSMC method[3] models the gas by some thousands of simulated molecules in a computer. The position coordinates and velocity components of each molecule are modified with time as molecules are followed through representative collisions and boundary interactions in simulated space. The flowfield was divided into a computational cell network as indicated in Fig. 3, which facilitates the choice of potential collision pairs and the sampling of the macroscopic flow properties. The dimensions of the cell should be small enough so that changes in flow properties across the cell are small. The flowfield measured 6 satellite diameters in the x direction and 2 diameters in the radial direction, with a maximum of 60 cells in the x direction and 30 in the radial direction. The satellite surface was assumed to be at the same temperature as the freestream and to reflect all incoming molecules diffusely, completely accommodated to the surface temperature. The ejected molecules entered the simulation in a uniform stream distributed over the surface having a specified number density, with velocities conforming to a Maxwellian distribution with a specified mean velocity in the negative x direction and a specified temperature. The properties of the ejected molecules could be varied as desired.

Two choices of the collision probabilities in the simulation were used, one corresponding to a constant total collision cross section and the other to a total collision cross section proportional to $g^{-0.3}$, which is appropriate for an inverse 13.5 power intermolecular potential. In both cases the differential collision cross section was taken as that for hard spheres. The combination of variable total cross section and hard sphere differential cross section has been proposed for direct simulation methods by a number of authors[4-6] and is generally called the "variable hard sphere model."[4] The total cross section for the variable hard sphere was chosen to make the theoretical viscosity coefficient, at the freestream conditions, the same as for the hard sphere molecules. There was little difference between the two total drag coefficients found with the two molecular models, so the hard sphere model was predominantly used.

Results

The Monte Carlo simulation was used to investigate different combinations of ejected jet velocity, temperature, number density, and mass and diameter of jet molecules, with a uniformly ejected stream. It was found that the number densities of both freestream and ejected molecules are greatest in the region very close to the disk surface for all cases considered. Since the collision frequency is proportional to the product of the number densities, this indicates that the collision zone is near the disk surface and is not far away as Stalker had first postulated.

The Monte Carlo simulations also confirm the drag results obtained from the analytical model; that is, the total drag on the disk, taking into account the reaction from the ejected molecules, is increased slightly by the upstream ejection. Figure 4 shows C_D for typical freestream conditions and different ejection speeds normalized by the drag coefficient obtained from a simulation with no upstream ejection C_{D0}. The drag coefficient with no ejection is less than the free molecular drag coefficient assumed for the analytical model since there is a compression of the gas in front of the disk producing a stagnation density approximately 20 times the freestream value. Consequently, the flow is transitional rather than free molecular and the Monte Carlo simulations account for this effect.

Both the analytical and numerical analyses indicate that upstream ejection using a uniform jet is unable to reduce the aerodynamic drag, and so different jet geometries were investigated using the simulation method. Two variations of diverging jets and a converging jet were simulated, and it was found that the reduction in total drag coefficients, if any, was minimal compared with the uniformly ejected stream.

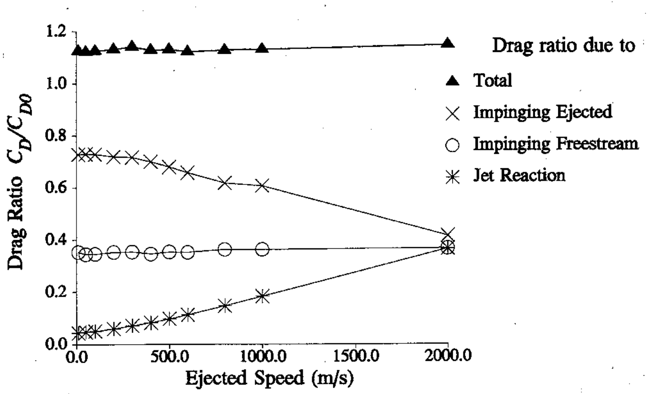

Fig. 4 Drag coefficients on circular disk incorporating molecular sweeping. Results from Monte Carlo simulation using hard sphere molecules and a freestream speed of 7.6 km/s, speed ratio of 16.5, Knudsen number of 1.6, $m_1 = m_2$, $T_1 = T_2 = T_S$, and $R_f = 1.25$; C_{D0} = drag coefficient for no ejection = 1.85 (from DSMC method using hard sphere molecules).

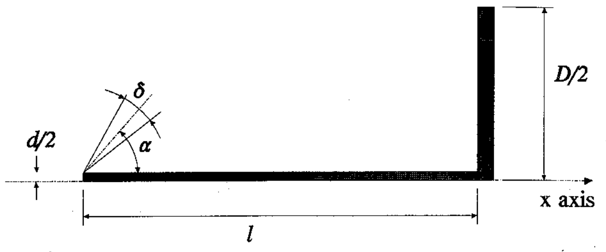

Fig. 5 Schematic of sting and point source of ejection. The arrangement is symmetric about the x axis.

Artificially Removing the Collision Zone (Incorporation of a Sting)

The distance of the collision zone from the surface determines the effectiveness of upstream ejection for drag reduction. If it is possible to artificially remove the collision zone, perhaps the ejection process would work. A sting equipped with a diverging nozzle is suggested that could telescope out in front of the satellite.

A first collision analysis applied to a point source of ejected molecules located on the x axis some distance from the satellite was undertaken, and it was found that a distance of four diameters ($4D$) from the disk surface results in negligible scatter onto the disk's surface. It is likely that a point source located further from the disk would result in little or no improvement because of the increased chance that further collisions would reflect a previously deflected molecule back into the satellite's path.

The sting was modeled for a DSMC simulation as a finite width beam of diameter d and length l, protruding from the front surface of the satellite (Fig. 5). The jet was simply modeled as a point source (in axisymmetric coordinates) located on the exterior of the sting. The angle of ejection α and the angle of divergence δ could be specified as desired.

Monte Carlo simulations were made with freestream conditions appropriate to a low-Earth orbit with an upstream Knudsen number of 1.6. The direction of ejection, velocity of ejection, and mass flow rate of ejected molecules were varied according to what was thought practicable to find a combination that provides the best results, that is, the least mass requirements to maintain the orbit velocity.

A reaction rocket for typical flight must balance the drag on the satellite to maintain the satellite's orbit velocity, that is,

$$m_r n_r V^2 A_r = \tfrac{1}{2} C_{D0} m_1 n_1 U_1^2 A \tag{2}$$

where C_{D0} is calculated from the DSMC method. For a low-Earth orbit, the freestream velocity U_1 is approximately 7.6 km/s, whereas exhaust velocities V are approximately 1.9 km/s. Therefore, putting $U_1/V \approx 4$, Eq. (2) becomes

$$m_r n_r V A_r = \tfrac{1}{2} C_{D0} m_1 n_1 U_1 A (U_1/V) \approx 2 C_{D0} m_1 n_1 U_1 A \tag{3}$$

The total mass flow rate to maintain flight for a satellite incorporating upstream ejection is comprised of the mass flow rate of ejected molecules and the mass flow rate for a reaction rocket to overcome the residual drag, which is the vector sum of the aerodynamic drag caused by molecules hitting the disk and the reaction from the ejection process. Note that the ejection may add to or subtract from the aerodynamic drag depending on whether the angle of ejection α in Fig. 5 is greater than or less than 90 deg. By similar reasoning that led to Eq. (3), the mass flow rate of a reaction rocket necessary to overcome the residual drag on the satellite is

$$m_r n_r V A_r = \tfrac{1}{2} C_D m_1 n_1 U_1 A (U_1/V) \approx 2 C_D m_1 n_1 U_1 A \tag{4}$$

The total mass flow rate for the upstream ejection system (ejected and reaction rocket) is then $m_2 n_2 U_2 A_2 +$

$2C_D m_1 n_1 U_1 A$. The fraction of mass required for sustained flight using upstream ejection compared with a typical reaction rocket is then

$$\frac{(m_2 n_2 U_2 A_2 / m_1 n_1 U_1 A) + 2C_D}{2C_{D0}} = \frac{R_f + 2C_D}{2C_{D0}} \tag{5}$$

The DSMC method was used to investigate various upstream ejection arrangements. For a sting length of $4D$, the optimal angles of ejection were found to be an ejection angle α of 10 deg and a divergence angle δ of 20 deg.

The velocity of the jet was varied from zero to a maximum velocity of 2 km/s, which is an estimate of the maximum velocity able to be achieved using a simple, controllable rocket thruster. The results represented in Fig. 6 were obtained using a constant mass flow rate ratio R_f of 1.25 and show that the drag due to impinging molecules on the disk surface increases as the ejected velocity increases. This increase, however, is more than compensated by the increase in thrust that is provided as the velocity is increased, and the lowest overall drag coefficient is achieved using the maximum ejected velocity.

The mass flow rate $m_2 n_2 U_2 A_2$ was varied by altering the number density of ejected molecules (Fig. 7). The velocity was held constant at 2 km/s. An ejected mass flow rate of approximately $1.9 m_1 n_1 U_1 A$ results in a total drag of zero and so provides the capability of maintaining flight solely with the upstream ejection system (that is, without a backward pointing reaction rocket), and the percentage of mass needed to maintain flight compared with a typical reaction rocket is reduced to just over 50%. Figure 8 shows the mean momentum vectors of the freestream molecules for a typical simulation with the sting incorporated both with and without upstream ejection. The radial divergence of the freestream molecules is clearly visible in Fig. 8a. This leads to a substantial reduction in the momentum of the freestream in the region close to the disk, resulting in a large decrease in the aerodynamic drag due to the freestream.

Varying the sting length ratio l/D confirmed that increasing the ratio l/D from 4 provides little advantage (Fig. 9). For shorter stings, however, the drag on the disk was found not to increase dramatically unless l/D was less than 2.0.

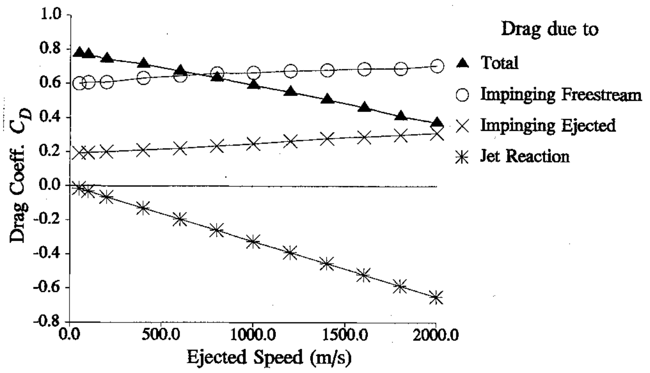

Fig. 6 Drag coefficients on circular disk incorporating sting vs ejected velocity. Results from Monte Carlo simulation using hard sphere molecules and a freestream speed of 7.6 km/s, speed ratio of 16.5, Knudsen number of 1.6, sting length of $4D$, α of 10 deg, δ of 20 deg, $m_1 = m_2$, $T_1 = T_2 = T_S$, and $R_f = 1.25$.

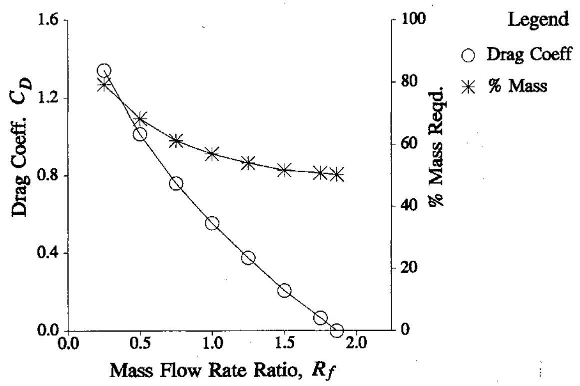

Fig. 7 Total drag coefficients and mass requirements for disk incorporating sting vs mass flow rate ratio R_f. Results from Monte Carlo simulations and Eq. (5) for an ejected velocity of 2 km/s. All other parameters as per Fig. 6.

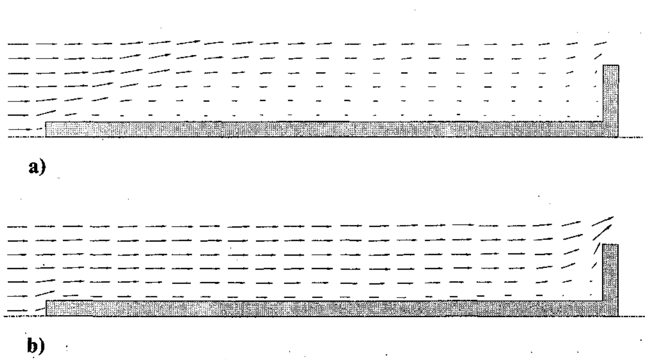

Fig. 8 Mean momentum vectors of freestream molecules a) with upstream ejection, $R_f = 1.75$ and $U_2 = 2$ km/s, and b) in the absence of upstream ejection, i.e., $R_f = 0$. All other parameters as per Fig. 6.

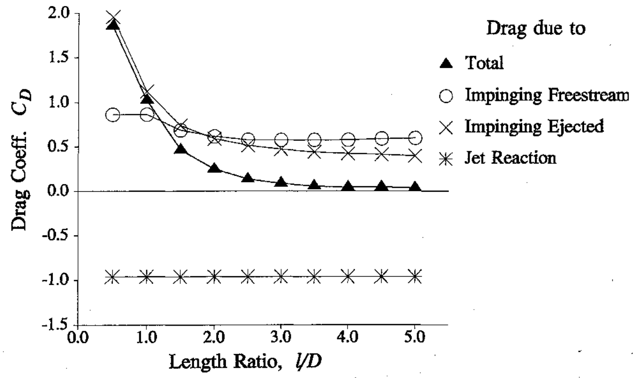

Fig. 9 Drag coefficients on disk incorporating sting vs length ratio of sting. Results from Monte Carlo simulation for an ejected velocity of 2 km/s and mass flow rate ratio of 1.85. All other parameters as per Fig. 6.

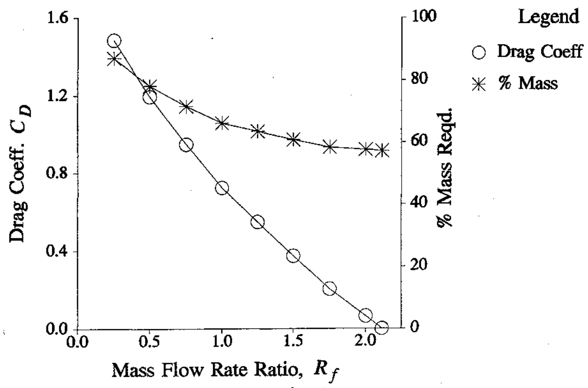

Fig. 10 Total drag coefficients and mass requirements for disk incorporating sting vs mass flow rate ratio R_f. Results from Monte Carlo simulations and Eq. (5) for a sting length of $2D$, α of 30 deg, δ of 10 deg, and ejected velocity of 2 km/s. All other parameters as per Fig. 6.

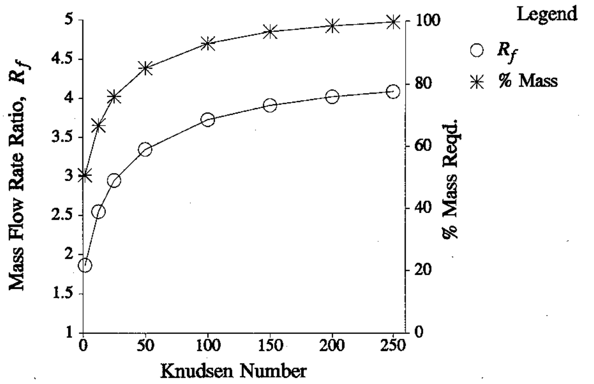

Fig. 11 Mass flow rate ratio R_f and mass requirements for disk incorporating sting vs freestream Knudsen number. Results from Monte Carlo simulations and Eq. (5) for a sting length of 4D and ejected velocity of 2 km/s. All other parameters as per Fig. 6.

Since it is desirable for structural reasons to use the short rather than the long sting, the best case using a sting l/D of 2.0 was then investigated. The optimum angles of ejection change for different sting lengths. For a length of 2D, the best results were obtained for a divergence angle δ of 10 deg centered on an ejection angle α of 30 deg. Such a small divergence angle may be difficult to obtain in practice, but the results are not sensitive to the divergence angle. For example, the difference between the fraction of mass required [Eq. (5)] using a divergence angle of 10 and 20 deg with an ejection angle of 30 deg is less than 1%. It was found (Fig. 10) that a mass flow rate ratio of 2.1 results in a net drag of zero, negating the need for a reaction rocket, and the total propellant mass needed to maintain flight is approximately 57%.

For a smaller satellite or a larger freestream mean free path, the effectiveness of the sting-mounted ejection process decreases. Different Knudsen numbers were simulated using the DSMC method and it was found (Fig. 11) that the mass required increases with increasing Knudsen numbers and is virtually the same as for a conventional reaction rocket system at a freestream Knudsen number of 250.

The primary collisions between the freestream and ejected molecules, which are the most important for the drag reduction, occur at high relative speeds. Comparison of the viscosity of air at the freestream temperature and at a temperature characteristic of these high-speed collisions indicates that the effective collision cross section will be reduced by approximately 35% compared with the characteristic cross section that determines the freestream Knudsen number. The hard sphere simulation results may then more accurately represent Knudsen numbers of the real gas that are slightly smaller than the quoted Knudsen numbers based on hard sphere representative molecules.

Conclusion

Direct upstream ejection from an entire face of a satellite surface or molecular sweeping as proposed by Stalker was found to be ineffective for reducing the net drag on a satellite in a low-Earth orbit. Molecular sweeping did not work as predicted because the primary collision zone always remained close to the satellite as the mass flow rate of ejected molecules was increased. However, the collision zone can be moved away from the satellite by using a sting arrangement pointing directly upstream from the surface of the satellite. Best results are obtained by ejecting molecules in the downstream direction so that they both clear incoming molecules from the satellite path and provide some forward thrust. Such an arrangement has been investigated by direct simulation of the molecular motions and collisions. The results indicate that a substantial mass saving of propellant can be achieved using this method. For the ideal case, where the sting is represented as a long beam ($l/D > 4$), a mass reduction of nearly 50% can be achieved. A more realistic sting (length equal to twice the satellite diameter, with a diameter of one-tenth of its length) was found to overcome atmospheric drag (without the incorporation of a conventional reaction rocket) using less than 60% of the mass of propellant expected to be used by conventional rockets. Any savings of fuel mass must be balanced against the mass of the sting and upstream ejection system, but in the best case our results show that the traditional propulsion system can be eliminated entirely. This requires the velocity of ejection to be 2 km/s, which is only obtainable with a chemical propulsion system, and may therefore restrict the use of such a scheme due to the possible contamination of the satellite by the ejected molecules.

Appendix: Mean Free Path of Upstream Ejected Molecules

To find λ_{21x}, the average distance a jet molecule travels in the negative x direction before colliding with a freestream molecule, we used an approach similar to that used by Whitfield.[7] The total number of collisions between pairs of hard sphere molecules of class 1 (freestream) and class 2 (ejected) per unit volume and unit time is given by

$$N_{12} = \int\!\!\int\!\!\int\!\!\int\!\!\int\!\!\int \pi g \sigma_{12}^2 f_1 f_2 \, du_1 \, dv_1 \, dw_1 \, du_2 \, dv_2 \, dw_2 \tag{A1}$$

The Maxwellian distribution functions for these two classes of molecules are

$$f_1 = \frac{n_1}{(2\pi R T_1)^{3/2}} \exp\{-(1/2RT_1)[(u_1 - U_1 \sin\beta)^2 + (v_1 + U_1 \cos\beta)^2 + w_1^2]\} \tag{A2}$$

and

$$f_2 = \frac{n_2}{(2\pi R T_2)^{3/2}} \exp\{-(1/2RT_2)[(u_2 + U_2)^2 + v_2^2 + w_2^2]\} \tag{A3}$$

The coordinate system used is the same as that represented in Fig. 1; that is, the u_i, v_i, and w_i are orthogonal velocity components with u_i directed inward and normal to the surface. For the derivation, the mean velocity U_1 is orientated at an angle β from the surface. An angle of 90 deg corresponds to the surface being aligned normal to the freestream. The velocity of ejection U_2 is defined for convenience as being positive in the negative x direction.

Nondimensionalizing by the variables defined in the nomenclature, and including Eqs. (A2) and (A3), Eq. (A1) becomes

$$N_{12} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{2\pi\sigma_{12}^2 n_1 n_2 (2RT_1)^{1/2}}{\pi^3} h(P) \times \exp\{-[(\xi_1 - \omega_1)^2 + (\eta_1 + \omega_2)^2 + \zeta_1^2 + (\xi_2 + \omega_3)^2 + \eta_2^2 + \zeta_2^2]\} d\xi_1 \, d\eta_1 \, d\zeta_1 \, d\xi_2 \, d\eta_2 \, d\zeta_2 \tag{A4}$$

where

$$h(P) = [(\xi_1 - \tau\xi_2)^2 + (\eta_1 - \tau\eta_2)^2 + (\zeta_1 - \tau\zeta_2)^2]^{1/2} \tag{A5}$$

and

$$P = (\xi_1, \eta_1, \zeta_1, \xi_2, \eta_2, \zeta_2) \tag{A6}$$

The mean collision rate of each class 2 molecule with a class 1 molecule is $\nu_{21} = N_{12}/n_2$, and the mean distance traveled in the negative x direction (directly upstream) by molecules of class 2 between successive collisions with those of class 1 is given by

$$\lambda_{21x} = \frac{|\bar{u}_2|}{\nu_{21}} = \frac{n_2|\bar{u}_2|}{N_{12}} \tag{A7}$$

where $|\bar{u}_2|$ is the mean speed of all class 2 molecules with $u_2<0$, that is, the mean speed of molecules traveling in the negative x direction. It is convenient to normalize this mean free path with respect to the mean free path of freestream molecules for collisions among themselves, which is $\lambda_{11} = [(2)^{1/2}\pi n_1 \sigma_{11}^2]^{-1}$. Letting $\sigma_1=\sigma_2$, Eq. (A7) can be written as

$$\frac{\lambda_{11}}{\lambda_{21x}} = \frac{N_{12}}{(2)^{1/2}\pi\sigma_{12}^2 n_1 n_2 |\bar{u}_2|} \tag{A8}$$

A first-order approximation to the collision rate N_{12} that appears in Eq. (A8) can be found by first expanding $h(P)$, Eq. (A5), in a Taylor series in its six independent variables about the point $Q=(\omega_1, -\omega_2, 0, -\omega_3, 0, 0)$, to obtain

$$\begin{aligned} h^{(1)}(P) &= S^* + (\omega_1/S^*)(\xi_1 - \omega_1 - \tau\xi_2 - \tau\omega_3) \\ &- (\omega_2/S^*)(\eta_1 + \omega_2 - \tau\eta_2) \\ &+ (\omega_3/S^*)(\tau\xi_1 - \tau\omega_1 - \tau^2\xi_2 - \tau^2\omega_3) \end{aligned} \tag{A9}$$

where

$$S^* = [(\omega_1 + \tau\omega_3)^2 + \omega_2^2]^{1/2} \tag{A10}$$

Then $h^{(1)}(P)$ is substituted into Eq. (A4) and the integration is performed. The resulting expression for N_{12} and the following expression for $|\bar{u}_2|$,

$$|\bar{u}_2| = \frac{(2RT_2)^{1/2}}{(\pi)^{1/2}}\left[\frac{\exp(-\omega_3^2)}{1+\mathrm{erf}(\omega_3)} + (\pi)^{1/2}\omega_3\right] \tag{A11}$$

which follows from Eq. (A3), can be substituted in Eq. (A8) to obtain

$$\begin{aligned} \left(\frac{\lambda_{11}}{\lambda_{21x}}\right)^{(1)} &= \frac{(2)^{1/2}}{4(\{\exp(-\omega_3^2)/[1+\mathrm{erf}(\omega_3)]\} + (\pi)^{1/2}\omega_3)} \\ &\times\Bigg(\left(\frac{\omega_1}{\tau S^*} + \frac{\omega_3}{S^*}\right)\{\exp(-\omega_1^2)[1+\mathrm{erf}(\omega_3)] \\ &+ \tau\exp(-\omega_3^2)[1+\mathrm{erf}(\omega_1)]\} \\ &+ \frac{S^*}{\tau}(\pi)^{1/2}[1+\mathrm{erf}(\omega_1)][1+\mathrm{erf}(\omega_3)]\Bigg) \end{aligned} \tag{A12}$$

The freestream Knudsen number is defined as $Kn=\lambda/D$, and so the distance that an ejected molecule travels upstream from a surface before undergoing a collision with a freestream molecule can be written as

$$\lambda_{21x} = KnD\left(\frac{\lambda_{21x}}{\lambda_{11}}\right)^{(1)} \tag{A13}$$

where $(\lambda_{21x}/\lambda_{11})$ is given by Eq. (A12).

References

[1]Stalker, R. J., "Upstream Ejection in Rarefied Flows—'Molecular Sweeping'," AIAA Paper 87-0403, Jan. 1987.

[2]Muntz, E. P., and Hanson, M., "Purging Flow Protection of Infrared Telescopes," *AIAA Journal*, Vol. 22, No. 5, 1984, pp. 696–704.

[3]Bird, G. A., *Molecular Gas Dynamics*, 1st ed., Clarendon Press, Oxford, England, UK, 1976.

[4]Bird, G. A., "Monte Carlo Simulation in an Engineering Context," 12th International Symposium on Rarefied Gas Dynamics, Paper 221, July 1980; also *Rarefied Gas Dynamics*, edited by S. S. Fisher, Vol. 74, Pt. 1, Progress in Astronautics and Aeronautics, AIAA, New York, 1981, pp. 239–255.

[5]Borgnakke, C., and Larsen, P. S., "Statistical Collision Model for Monte-Carlo Simulation of Polyatomic Gas Mixture," *Journal of Computational Physics*, Vol. 18, Aug. 1975, pp. 405–420.

[6]Erofeev, A. I., and Perepukhov, V. A., "Hypersonic Rarefied Flow about a Flat Plate by the Direct Simulation Method," *Proceedings of the 11th International Symposium on Rarefied Gas Dynamics*, Pt. 1, Paris, 1979, pp. 417–421.

[7]Whitfield, D. L., "Mean Free Path of Emitted Molecules and Correlation of Sphere Drag Data," *AIAA Journal*, Vol. 11, No. 12, 1973, pp. 1666–1670.